

PID: \_\_\_\_\_

Last Name, First Name: \_\_\_\_\_

Section: \_\_\_\_\_

Homework 3  
Math 183, UCSD, Fall 2017  
Due on Friday 27th, October 12:50pm  
Staple pages together

## Exercise 1

Lynndyl is a pretty small city in Utah, with only 106 inhabitants. Seventy-one percent of the registered voters favor their incumbent mayor in her bid for re-election. If only twenty-four citizens go to the polls:

1. What is the probability that the race ends in a tie?

2. What number of voters do you expect to have voted for her challenger?

## Exercise 2

1. Do you think the following set of data is likely to have been generated by a random variable  $X$  that has the geometric distribution  $P(X = k) = \left(\frac{4}{5}\right)^{k-1} \times \frac{1}{5}$ , ( $k = 1, 2, \dots$ )? Explain.

|   |   |   |   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|---|---|---|
| 2 | 8 | 1 | 2 | 2 | 5 | 1 | 2 | 8 | 3 |
| 5 | 4 | 2 | 4 | 7 | 2 | 2 | 8 | 4 | 7 |
| 2 | 6 | 2 | 3 | 5 | 1 | 3 | 3 | 2 | 5 |
| 4 | 2 | 2 | 3 | 6 | 3 | 6 | 4 | 9 | 3 |
| 3 | 7 | 5 | 1 | 3 | 4 | 3 | 4 | 7 | 6 |

2. With your cellphone, transmission errors occur with a rate of approximately 2.5 per ten seconds. What is the probability that more than two errors will occur during the next half-minute?

## Exercise 3

The Biology department bought a brand new spectrophotometer. This device was sold with a service contract that ensures 4 free breakdown repairs from a technician. A breakdown of such a device occurs, on average, three times a year. These breakdowns are independent from each other.

1. Write explicitly a model and a random variable that describes the time elapsed until the first intervention of a technician.

2. Write explicitly a random variable describing the time for the service contract to be fulfilled.

3. What is the average time it will take for the service contract to be fulfilled?

## Exercise 4

For an adult at rest, the time between two consecutive eye blinks (inter-blinking time), expressed in seconds, is well-modeled by a continuous random variable  $X$  having density

$$f(x) = \begin{cases} c \cdot x^3 & 0 \leq x \leq 5 \\ 0 & \text{otherwise,} \end{cases}$$

where  $c$  is a positive constant.

1. What value must  $c$  be if  $f(x)$  is to be a valid density function?

2. What is the probability someone does not blink for more than 3 seconds?

3. What is the probability someone's inter-blinking time is exactly 1.67 seconds?

4. What is the expected time between two consecutive blinks?

5. What is the standard deviation of this duration?

6. During a medical experiment, a scientist tells you she observed an inter-blinking time among the shortest 5%. Find the longest inter-blinking time she could have observed.

## Exercise 5

The average daily temperature in October in Boston is  $61^{\circ}\text{F}$  with a standard deviation of  $5^{\circ}\text{F}$ . These temperatures closely follow a normal distribution.

1. What is the probability of observing a  $67^{\circ}\text{F}$  temperature or higher in Boston during a randomly chosen day in October?

2. How warm are the hottest 20% of the days (days with highest average high temperature) during October in Boston?

We use the following equation to convert  $^{\circ}\text{F}$  (Fahrenheit) to  $^{\circ}\text{C}$  (Celsius):

$$C = \frac{5}{9}(F - 32).$$

4. Write the probability model for the distribution of temperature in  $^{\circ}\text{C}$  in October in Boston.

5. What is the probability of observing a  $20^{\circ}\text{C}$  (which roughly corresponds to  $68^{\circ}\text{F}$ ) temperature or higher in October in Boston? Calculate using the  $^{\circ}\text{C}$  model from question 4.

6. Estimate the IQR of the temperatures (in  $^{\circ}\text{C}$ ) in October in Boston.